COMMUNITY OF PHYSICS

1st Workshop on Vector Calculus

Full Marks: 80
Time: 2 Hours and 30 Minutes

1. A company that manufactures dog food wishes to pack the food in closed cylindrical tins. What should be the dimensions of each tin if each is to have a volume of 128π cm^3 and the minimum possible surface area? (05)

2. Find any two of the following integrals, where n and m can be any integer numbers: (03+03)
   a. $\int_0^{2\pi} \sin(mx)\cos(nx)\,dx$
   b. $\int_0^{2\pi} \sin(mx)\sin(nx)\,dx$
   c. $\int_0^{2\pi} \cos(mx)\cos(nx)\,dx$
   Find all possible cases, where n=m=0, n=m≠0 and n≠m.

3. Computer Graphics System is an example of five dimensional vector space. It requires two independent numbers to store the information of the position of a pixel and three independent numbers to store the information of the color (Red, Blue and Green) of the pixel. In a 32bit color system, each color can take any value from 0 to 255. Find a convenient linear transformation on the vector space that performs an inversion operation on the color system. [Hint: In inversion, colors of every pixel become

inverted. For example, a color assigned for a pixel as (100, 150, 55) will become (155, 105, 200).] (10)

4. Find the divergences and curls of the following vector fields: (05)
   a. $\boldsymbol{A}(\boldsymbol{r}) = xy^3\hat{\boldsymbol{x}} + zxy\hat{\boldsymbol{y}} + xy^5\hat{\boldsymbol{z}}$
   b. $\boldsymbol{B}(\boldsymbol{r}) = e^{yz}\hat{\boldsymbol{x}} + \ln(2zx)\,\hat{\boldsymbol{y}} + \sec(xy)\,\hat{\boldsymbol{z}}$

5. A parabolic surface is defined as, $z = x^2 + y^2$. Find a unit vector, perpendicular to the surface, at (3, 2). (07)

6. A particle follows a trajectory $\boldsymbol{r}(t) = 3m\cos\omega t\,\hat{\boldsymbol{x}} + 2m\sin\omega t\,\hat{\boldsymbol{y}} + 6ms^{-1}t\hat{\boldsymbol{z}}$. Find the curvature and torsion of the trajectory. (10)

7. Find the volume bounded by the surfaces $z = 9 - 2x^2 - y^2$ and $z = 2$. (08)

8. Using Maxwell's second equation $\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$, Maxwell's fourth equation $\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0\epsilon_0\frac{\partial \boldsymbol{E}}{\partial t}$ and continuity equation $\boldsymbol{\nabla} \cdot \boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0$, directly show that, $\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$ and $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$. (07)

9. An infinite straight wire of radius 2mm is carrying a steady current of 5A. Find the magneto-static total energy of the current configuration inside the region bounded by the surfaces $z = 0$ and $z = 5$. (15)

10. Prove that, $\boldsymbol{\nabla}(\boldsymbol{A}.\boldsymbol{B}) = \boldsymbol{A} \times (\boldsymbol{\nabla} \times \boldsymbol{B}) + \boldsymbol{B} \times (\boldsymbol{\nabla} \times \boldsymbol{A}) + (\boldsymbol{A} \cdot \boldsymbol{\nabla})\boldsymbol{B} + (\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{A}$. (07)