COMMUNITY OF PHYSICS

1st Workshop on Classical Electromagnetism

Full Marks: 100
Time: 2 Hours and 30 Minutes

1. Find the divergences and curls of the following vector fields. (08)
   a. $\boldsymbol{A}(\boldsymbol{r}) = xy^2\hat{\boldsymbol{x}} + zcos(xy)\hat{\boldsymbol{y}} + y^5\hat{\boldsymbol{z}}$
   b. $\boldsymbol{B}(\boldsymbol{r}) = e^{zx}\hat{\boldsymbol{x}} + \ln(xy)\,\hat{\boldsymbol{y}} + \sin(yz)\,\hat{\boldsymbol{z}}$

2. Prove the following vector identities using Levi-Cività symbol. (08)
   a. $\boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{B}) = \boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A}) - \boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$
   b. $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{A}) = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{A}) - \nabla^2\boldsymbol{A}$

3. A uniformly (and invariant) dense solid sphere of radius $2\ cm$ starts to expand just after adding a charge of $4\ C$ uniformly into it. After expansion its radius is $10\ cm$. Find the work-done by the electric field in the expansion process. (20)

4. Point charges of $3\times10^{-9}\ C$ are situated at each of three corners of a square whose side is $15\ cm$. Find the electric field at the vacant corner of the square. (10)

5. Given that the dielectric strength (i.e., the electric field above which the air becomes conducting) is $3 \times 10^6\ V/m$, what is the highest possible potential of an isolated spherical conductor of radius $10\ cm$. (15)

6. Find the magnetic vector potential for a current carrying circular loop of radius $5\ cm$ and current $2\ A$ at point $4\ cm$ above the center of the loop. (10)

7. A long coaxial cable carries current $2\ A$. Current flows down the surface of the inner cylinder of radius $2\ mm$ and back the outer cylinder of radius $4\ mm$. Find the energy stored in a section of length $3\ cm$. (15)

8. Calculate the Fourier coefficients for the given *continuous periodic* function $f(t)$, where T is the temporal period of the function.

$$f(t) = \begin{cases} 1 - (t-1)^2; 0 < t < \frac{T}{2} \\ (t-3)^2 - 1; \frac{T}{2} < t < T \end{cases}$$ (14)