COMMUNITY OF PHYSICS

1st Workshop on Differential Equation

Time 2:00 hours Full Marks 50

1. Solve the following differential equation subject to the given initial condition:
   a. $(1-x^2)\frac{dy}{dx} - 2xy = 0;\ \ y(0) = 1$
   b. $x\frac{dy}{dx} + (2x+1)y = e^{-2x};\ \ y(1) = \frac{1}{2}e^{-2}$
   c. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0;\ \ y(0) = 3;\ \frac{dy}{dx}(0) = 2$
   d. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0;\ y(0) = -3;\ \frac{dy}{dx}(0) = -1$
   e. $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 10y = 0;\ y(1) = 5;\ \ \frac{dy}{dx}(1) = 4$
2. A fossilized bone is found to contain one thousandth of the C-14 level found in living matter. Estimate the age of the fossil. Half-life of C-14 is approximately 5600 years.
3. A capacitor of capacitance $200\,\mu$ F is initially charged at 20V and then discharged through a resistor of resistance $50\text{k}\Omega$. What will be the voltage across the resistor after 8 seconds?
4. Find the Fourier coefficients of $f(x) = \sin^3 x + \cos^2 x$.
5. Plot the phase space trajectory of a mass-spring system of mass $m = 2mg$ and $k = 20N/cm$ using conservation of energy. It's total energy $E = 100J$
6. An undamped oscillator has a time period $\tau = 5ms$. Now I add a small damping and its period changes to $\tau = 5.005ms$. Then I add a driving force of amplitude $5cm$ and of variable frequency. Find the maximum amplitude of this damped driven oscillator. Find the quality factor.