

# COMMUNITY OF PHYSICS

## 1<sup>ST</sup> WORKSHOP ON CLASSICAL MECHANICS: FROM NEWTON TO LAGRANGE

FULL MARKS: 100

TIME: 2 HOURS AND 30 MINUTES

1. A student kicks a frictionless puck with initial speed 10 m/s, so that it slides straight up a plane that is inclined at an angle  $15^\circ$  above the horizontal. **(a)** Write down Newton's second law for the puck and solve to give its position as a function of time. **(b)** How long will the puck take to return to its starting point? (07)
2. A geostationary satellite named Syncom 2 is orbiting at a height of 30,000 km from earth surface. A GPS device that is inside a moving car of speed 30 ms<sup>-1</sup> sends a signal to Syncom 2. Syncom 2 sends back another signal as soon as receiving the signal. How much distance does the car travel in between sending and receiving the signal? (05)
3. A charged particle of mass  $m$  and positive charge  $q$  moves in uniform electric and magnetic fields,  $\mathbf{B}$  pointing in the  $y$ -direction and  $\mathbf{E}$  in the  $z$ -direction. **(a)** Write down Newton's second law for the charged particle and solve to give its velocity as a function of time. **(b)** For which initial velocity the charged particle will remain undeflected? **(c)** Integrate the expression of velocity, using a suitable choice of origin, to give the particle's position as a function of time. (10)
4. I flick a tiny metal pellet with diameter  $D=0.2$  mm, density  $\rho=16$  g/cm<sup>3</sup> and velocity  $\mathbf{v}=1$  m/s at  $45^\circ$  from horizontal plane. Find its horizontal range. (08)

5. A large Foucault pendulum such as hangs in many science museums can swing for many hours before it damps out. Taking the decay time to be about 8 hours and the length to be 30 meters, find the quality factor. [Hint: Quality factor  $Q$  is defined as the ratio of half of the natural angular frequency per unit decay parameter,  $Q = \frac{\omega_0}{2\beta}$ .] (08)
6. Calculate the Fourier coefficient for the given periodic function, where  $T$  is the temporal period.  $f(t) = \begin{cases} 1 - t; & 0 < t < \frac{T}{2} \\ t - 1; & \frac{T}{2} < t < T \end{cases}$  (07)
7. Find the gravitational field at the center of a ring of mass  $M$  and radius  $R$ . (07)
8. An earth satellite is observed at perigee to be 250 km above the earth's surface and travelling at about 8500 m/s. Find the eccentricity of its orbit and its height above the earth at apogee. [Hint: The earth's radius is  $R_e = 6.4 \times 10^6$  m.] (08)
9. Consider again the Foucault pendulum described in problem 4. The length of its string is 30 meter and the mass of its bob is 2 kg. This heavy mass of bob reduces the decay parameter appreciably. Find its equation of motion at  $45^\circ$  latitude. Solve the equation of motion to give the bob's position as a function of time. (25)
10. Design a variable capacitance FM radio receiver that can operate between 88 mega Hertz and 108 mega Hertz. The variable capacitor you found in the market has a maximum capacitance of  $0.22 \mu\text{F}$  and a minimum capacitance of  $0.20 \mu\text{F}$ . What will be the minimum value of circuit resistance to receive signals of 0.1 MHz separation? (15)